

Model 8: World Dynamics

Narrative

Modeling growth and the limits to growth have been the central theme of all of the models. Hunter-gatherers were limited by the availability of their prey but, in the end, improved hunting technology often led to prey extinction. Farming greatly increased the number of humans that could be supported by a given area of land, providing a surplus that supported a growing number of non-farming elite. But even in agricultural systems with technological progress, human population growth closely tracked available resources as the producers and elites battled back and forth as exemplified by Peter Turchin's model of the rise and fall of civilizations.

With the advent of the industrial revolution (or the culmination of the agricultural conflagration as we prefer to think of it), technology began to progress faster than population growth. It appeared that economic growth per capita might continue on indefinitely. Robert Solow correctly identified that the primary driver responsible for the remarkable post-industrial revolution growth was not the increase in capital, as most economist had assumed, but rather advances in technology that increased production efficiency of each unit of capital.

What Solow's model inherently assumed, however, was that growth could continue on indefinitely. In the 1960's, a growing number of scientist increasingly questioned the economist's assumption that technological developments would continue leading to economic growth. Rather, these scientists suggested, humanity's continued economic growth would press against planetary environmental limits, perhaps irreversibly damaging the carrying capacity of the Earth, our only home. The time was ripe for a dynamic model that included planetary resource limits and the impact of human pollution. Enter Jay Forrester.

Forrester, an electrical engineer who spent his career at MIT, is noted for his pioneering development of random-access memory in early computers, and also for launching *system dynamics*, the study of dynamic systems using computer model simulations. His two lifetime achievements were related, as capable computers were required to run the system dynamics models.

His first 1961 book, *Industrial Dynamics*, used simulations similar to those presented in this module to model industries, including dynamic changes in their supply chains. Forrester noted in his 1968 book, *Principles of Systems*, that dynamic systems often produced counterintuitive results—that cause-and-effect relationships often operated in the opposite direction of what one would expect. One of the most important functions of models is to train our intuitions about how processes work. In his 1971 book, *Urban Dynamics*, Forrester developed a model of a city and its supporting systems. As might have been expected, some of the results of his simulations were counterintuitive. Since they opposed current wisdom, his book stirred up considerable political controversy.

However, it is his 1971 book, *World Dynamics*, that is of the most interest to us (the second, 1972, edition which is available as a PDF). This historic model was developed, in the first weeks of July 1971, by Forrester and his colleagues (including Dennis and Donella Meadows), in preparation for a visit to MIT by members of the Club of Rome—a group of influential citizens concerned with the future of humanity and the planet. After their visit, the Club of Rome funded a one-year effort at MIT that culminated in a refined model, World-3 and the 1972 book, *The Limits to Growth*, by Donella Meadows,

Dennis Meadows, Jorgen Randers, and William Behrens. This book, according to Wikipedia, has sold 30 million copies in 30 languages. The World-3 model has been refined over the years with updated data, and has resulted in three additional books, the latest being the 2012 book by Jorgen Randers, *2052: A Global Forecast for the Next Forty Years*.

In this module, you can exercise the historic and pioneering *World Dynamics* model that led to a significant change in humanity's view of its potential future on planet Earth.

The human population sector of this model and the pollution sector (via greenhouse gas emissions) have received much attention since 1970. Paul Ehrlich's 1968 book, *The Population Bomb*, called attention to the very rapid world population growth rate at that time and predicted that it would cause huge problems of resource depletion and pollution. Many of Ehrlich's specific predictions have not come to pass, in part because the threat of population growth led to strenuous R&D work in agriculture and industry to reduce pollution and increase the efficiency of industry and agriculture. But also because "prediction is hard, especially the future," a remark, variously attributed to Mark Twain, Niels Bohr, and Yogi Berra, that we all need to take on board. The world is a complex, uncertain system, and people who make predictions are almost certain to be humbled!

Ehrlich was right in his general claim that rising human populations were a problem. The discussion has gotten a lot more sophisticated in the ensuing half century but he, like-minded authors, and his critics laid down the broad outlines of the debate. Humans have certainly become a global biogeochemical agent comparable in magnitude to other major processes like photosynthesis, solar radiation, volcanic production of CO₂, and the sedimentary storage of carbon. If we do not divert a significant portion of our economic output to investment in managing global commons resources, and make technological progress to assist this management, we are highly likely to meet one or another Ehrlichian catastrophe.

Since 1970, human induced global warming has become the single most widely discussed potentially catastrophic human impact on the environment. The theory of the CO₂ regulation of global temperature via the greenhouse effect was developed in the 19th century. In 1958, Charles Keeling, then at the University of California Scripps Institution of Oceanography, established an observatory at the 11,000 foot level of the tall, isolated, barren Mona Loa volcano, on the Big Island of Hawai'i to measure atmospheric CO₂ concentrations. This location is minimally subject to local pollution by anthropogenic CO₂ and CO₂ uptake by local photosynthesis.

By the mid-1970s, it became clear that the global concentration of CO₂ was increasing steadily and rapidly. Simple calculations suggested that the observed rate of increase might have a major impact on the radiation balance of the earth. This initiated an intensive investigation of the earth's carbon cycle, paleoclimate investigations to see if ancient variations in CO₂ concentrations had played an important role in climate regulation (they had), launched an R&D push to develop solar, wind, biomass, and other alternatives to fossil fuels for energy production, and funded a sizeable program to construct complex coupled, ocean-atmosphere climate simulation models.

These models grossly violate the KISS principle. The pioneering climate modeler at Stanford University, Stephen Schneider (d. 2010) criticized them for this, observing that: (1) the different global climate models (GCMs), of which there are about 30, give quite different predicted global warming for a benchmark scenario in 2100, (2) the current models cannot take into account many of the important feedbacks in the climate system, and (3) models that do obey the KISS principle give you the same basic answer (approximately the mean of the predictions of the rather divergent complex models). Schneider

was quite pessimistic that GCMs could improve fast enough to usefully contribute to the policy debate. He advocated a modeling strategy aimed at trying to better understand the uncertainties in the climate system. In particular, are there any feedback processes that could give nasty surprises we really want to avoid or pleasant ones we might want to encourage.

Policy makers are stuck with these basic facts: (1) The CO₂ kick humans are delivering to the climate system is very significant, enough to raise global temperatures to something like the hothouse condition of 70 million years ago; and (2) the climate system is far too complex and too laced with non-linear feedbacks for scientists to produce precise long-term predictions of any of the details. Thus, the problem is the need to make momentous decisions under conditions of high uncertainty. In other words, as is common, life on a warming planet is rather more like a crazy adventure than many of us would like.

One aspect of the situation has turned out rather better than anticipated: population itself. Since 1970, most of the world has begun to follow the West in undergoing a demographic transition to small families. A number of countries have negative population growth rates and, if current trends continue, more will follow. But the demographic transition has not really gotten us around the corner. In a debate in 1970, Ehrlich, John Holdren, and a critic of theirs, Barry Commoner, came up with the thought equation $I = P \times A \times T$, where I signifies environmental impact, P is population, A is affluence, and T is technology. Some portion of the impact of people is due to the minimum needed for subsistence, but in many countries people consume much more than the subsistence minimum. In many more countries, including some that have rapidly growing economies and large populations, people aspire to rich country levels of affluence. If Ehrlich were writing today instead of in 1968 his title would be *The Affluence Explosion*. In terms of our models we can imagine that people have grown very big, so that the average person in rich countries has something like 10 times the ecological impact of the average person living in a subsistence economy.

The concept of *sustainability* has become important in recent years. The question is: how much should we invest in reducing impacts on the environment? For example, in the case of CO₂ emissions, one way to frame the question is how much should we tax each ton of CO₂ industry produces? Raising the cost of CO₂ emissions will motivate manufacturers to invest in R&D to reduce CO₂ emissions, for example by reducing the use of fossil fuels for moving people and goods. We might also want to use the revenue from such taxes to support technological or social innovations that are environment friendly, such as supporting research into minimizing the use of environmentally damaging products such as cement, or promoting the substitution of labor intensive products for resource intensive ones in our consumption bundles. For example, if people decided it is more important to buy original art than gasoline or jet fuel, that would shrink our carbon footprint. It would turn refinery workers into sculptors turning all those interestingly shaped metal tanks and pipes from the refinery into Watts Towers for the masses (https://en.wikipedia.org/wiki/Watts_Towers).

The tradeoffs involved in such calculations are easy to see but hard to calculate in an uncertain world. For example, if we assume that people in the future will be richer than those in the present (a conventional assumption for many economists), it follows that we should not burden people in the relatively poor present with high carbon taxes. Let our wealthy grandchildren pay more when their time comes! On the other hand, if you assume that we are going to leave our grandchildren with a hot world with diminished biodiversity that really can't be fixed, they will be poorer than we are. If so, we ought to pay a high carbon tax to spare them that misery! We can't know exactly how much damage a ton of CO₂ released today will do to the grandkid's environment. We don't know how much our grandkids will value

a cool climate and biodiversity. Perhaps they will be happy to live in high tech climate controlled pods and will think that their grandparents' obsession with biodiversity and an unpolluted outdoors are quaintly old fashioned.

Further Reading

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- Nordhaus, W. D. (2017). "Revisiting the social cost of carbon." *Proceedings of the National Academy of Sciences* 114(7): 1518-1523.
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- https://en.wikipedia.org/wiki/2052:_A_Global_Forecast_for_the_Next_Forty_Years
- https://en.wikipedia.org/wiki/Ecological_footprint
- https://en.wikipedia.org/wiki/Jay_Wright_Forrester
- https://en.wikipedia.org/wiki/The_Limits_to_Growth
- https://en.wikipedia.org/wiki/The_Population_Bomb
- https://en.wikipedia.org/wiki/Paul_R._Ehrlich
- https://en.wikipedia.org/wiki/Greenhouse_gas
- https://en.wikipedia.org/wiki/Charles_David_Keeling
- https://en.wikipedia.org/wiki/Stephen_Schneider
- https://en.wikipedia.org/wiki/Intergovernmental_Panel_on_Climate_Change
- https://en.wikipedia.org/wiki/I-3D_PAT
- <https://en.wikipedia.org/wiki/Sustainability>
- <https://en.wikipedia.org/wiki/Uncertainty>
- https://en.wikipedia.org/wiki/William_Nordhaus

White Box Graphical Model

The original World Dynamics model was executed on the 1970s DYNAMO computer simulation program developed by Jay Forrester and his colleagues. It was translated to the modern Stella program by Diana Fisher, and is shown in the figure below. This unit provides the Forrester model exactly as Fisher reconstructed it in Stella.

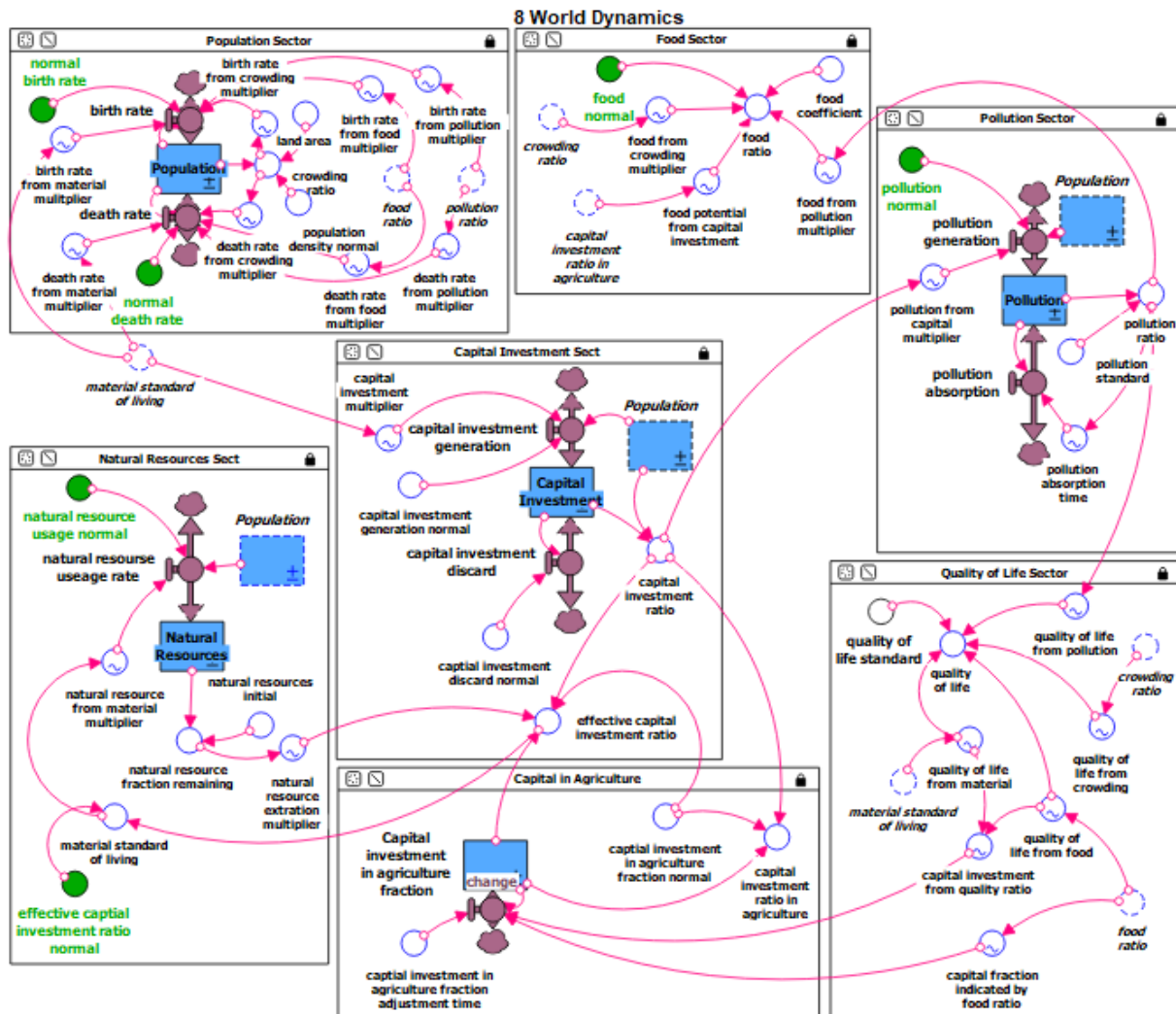


Fig 8-1: World Dynamics Visual Flow Diagram

The model is described in great detail in *World Dynamics*. This book is available online and can be downloaded and studied by those interested in the details of the model at:

https://monoskop.org/File:Forrester_Jay_W_World_Dynamics_2nd_ed_1973.pdf

This is a rather complex model compared to the others we have introduced in this course. Even so, it lacks one of the key features that characterizes human history, technological progress. As a consequence, it behaves a little like our predator-prey model and our hunting without dynamic technology model. The pollution submodel makes us something like predators on the environment. Without dynamic technology, this model cannot grow towards infinity the way our farming with dynamic technology model can. An interesting exercise would be to run this model for a thousand years or more to see what its long-term behavior is. Still, the common lesson of all of these models is that models with interesting stable states for humans and their support base are hard to build. It is as if humans are an inherently unstable species.

To stabilize human systems would seem to require deliberate stabilizing interventions. In principle, this is quite possible. Normally, airplanes are designed to be stable, to fly straight and level unless the pilot wants it to be otherwise. However such planes are not very nimble. Modern fighter planes are deliberately designed to be unstable so that they can be radically maneuvered in combat. Human pilots cannot fly such fighters without the aid of a flight control computer that essentially acts as a pilot with

super-fast and accurate reflexes to correct unstable behavior so that the pilot can fly the unstable beast straight and level, if required, and yet maneuver radically in reasonable safety if that is what is required.

What people who study these models typically advocate are public policy initiatives that act like a flight control computer to stabilize an otherwise unstable system. Conservatives, who often dislike interventionist measures seem to be saying that there is a naturally stable human system. If you want to be lionized in that community, build a model of the human system that is nice and stable! It would be a great contribution to the most important political debate in the modern world.

This model might also lead you to reflect on the KISS principle. Scientists and engineers often struggle with each other over model complexity. Engineers often build models of complex machines like airplanes that work quite well. Witness flight simulator programs. However, engineers have the luxury of building their own systems, introducing nonlinear feedbacks only when they want to, and using components with tightly engineered specs. Scientists must deal with natural systems with whatever nonlinearities nature throws up and with components whose parameters are uncertain, sloppy, and changing with time. They typically depend upon the qualitative insights from quite simple models and expect to modify them as experience dictates.

Black Box Model

An interface to the model has been developed so the original model can be exercised online. Five key independent variables, shown in green in the figure above, are adjustable.

To run this model from this black box perspective, bring it up at

<https://exchange.iseesystems.com/public/cherylgenet/world-dynamics>

You should see what appears in Figure 8-2.

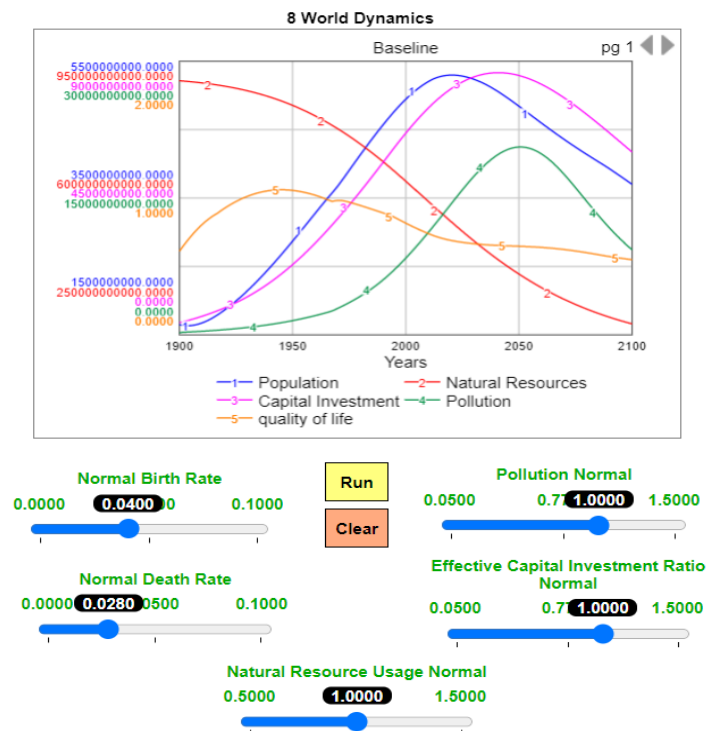


Figure 8-2: World Dynamics model interface.

Please note **pg 1** in the upper right corner. Click on it to bring up a second set of plots **pg 2** as shown below:

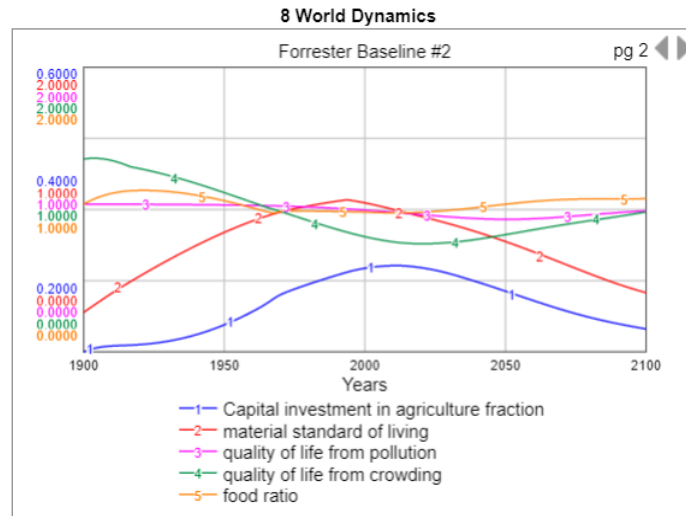
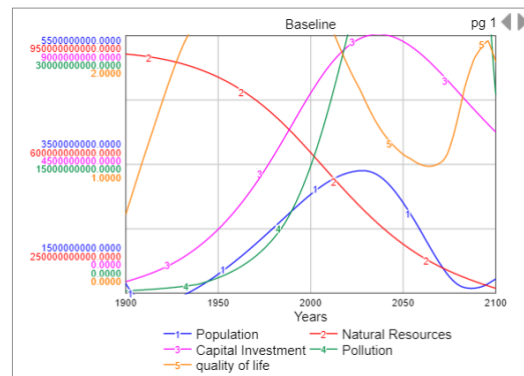
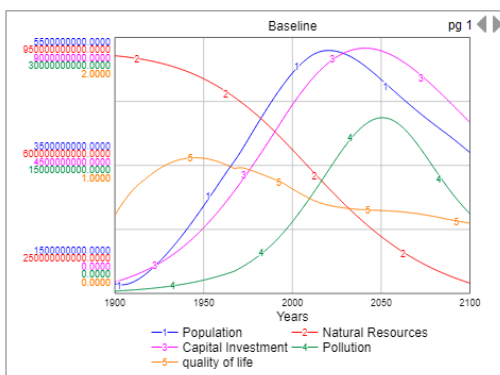


Figure 8-3: Second plot screen.

Also note that the time span is from 1900 to 2100. Forrester and his colleagues wanted to demonstrate that their model could replicate historic data to some extent and that the model could then project these historic trends off into the future. Although almost 50 years have passed since the model was developed, we stuck with the model's original timeline of 1900 to 2100.

Exercise: Lower the Birth Rate

Clear the model so it will set the default values of the independent variables (the sliders) and click *Run*. You should see the plots on the left, below. Now cut the **Normal Birth Rate** in half from 0.04 to 0.02. This is what we might expect if the *demographic transition* seen in many older industrial societies and China spread to the rest of the world. You should see the plot on the right below. What a huge change!



What is happening? As expected, the **Population-1-** is much lower. **Capital Investment-3-** stays about the same, as does the use of **Natural Resources-2-**, while **quality of life-5-** soars, perhaps not unexpected, as the same investment and resources are being expended on fewer people. Somewhat counterintuitively, **Pollution-4-** soars. Presumably, it is the growing, out of control pollution that

eventually drives the population and everything else down as the planet crashes and, in the end, starts a recovery. Perhaps the model assumes that the increased **Capital Investment** and **Natural Resources** on a per person will all be invested in improving **quality of life-5** with larger houses, flying to other countries for vacations, increased air conditioning in warmer climates, etc. Given what we've seen transpire over the past 50 years, this may not be a bad assumption!

Conclusion

It has been a long journey from early hunter-gatherers to farmers, civilizations, the rise of the modern world and our present encounter with planetary limitations. We hope that, between the narratives and the models, you have gained some insight into the dynamic nature of the evolutionary history of humankind and the role of models in scientific exploration and explanation. We encourage you to pursue the further reading references in areas that interest you.

Appendix / Stella Top Level Model Code

A Stella model is created by connecting the graphical elements and entering information in the Stella GUI interface. Once everything is connected and entered, Stella automatically creates the "top level code." This code provides a good check on whether or not the Stella model is what you really intended, and can be very useful in trouble shooting models that are not providing reasonable results or don't seem to be working at all.

Top-Level Model:

$\text{Capital_Investment}(t) = \text{Capital_Investment}(t - dt) + (\text{capital_investment_generation} - \text{capital_investment_discard}) * dt$

INIT Capital_Investment = 0.4e9 {capital units}

INFLOWS:

capital_investment_generation =

Population*capital_investment_generation_normal*capital_investment_multiplier {capital units/year}

OUTFLOWS:

capital_investment_discard = Capital_Investment*capital_investment_discard_normal {capital units/year}

$\text{Capital_investment_in_agriculture_fraction}(t) = \text{Capital_investment_in_agriculture_fraction}(t - dt) + (\text{change}) * dt$

INIT Capital_investment_in_agriculture_fraction = 0.2 {no units}

INFLOWS:

change = ((capital_fraction_indicated_by_food_ratio*capital_investment_from_quality_ratio)- Capital_investment_in_agriculture_fraction)/capital_investment_in_agriculture_fraction_adjustment_time {no units/year}

$\text{Natural_Resources}(t) = \text{Natural_Resources}(t - dt) + (- \text{natural_resource_usage_rate}) * dt$

INIT Natural_Resources = natural_resources_initial {natural resource units}

OUTFLOWS:

natural_resource_usage_rate =

Population*natural_resource_usage_normal*natural_resource_from_material_multiplier {natural resources units/year}

$\text{Pollution}(t) = \text{Pollution}(t - dt) + (\text{pollution_generation} - \text{pollution_absorption}) * dt$

INIT Pollution = 0.2e9 {pollution units}

INFLOWS:

$\text{pollution_generation} = \text{Population} * \text{pollution_normal} * \text{pollution_from_capital_multiplier}$ {pollution units/year}

OUTFLOWS:

$\text{pollution_absorption} = \text{Pollution} / \text{pollution_absorption_time}$ {pollution units/year}

$\text{Population}(t) = \text{Population}(t - dt) + (\text{birth_rate} - \text{death_rate}) * dt$

INIT Population = 1.65e9 {people}

INFLOWS:

$\text{birth_rate} =$

$\text{Population} * \text{normal_birth_rate} * \text{birth_rate_from_crowding_multiplier} * \text{birth_rate_from_food_multiplier} * \text{birth_rate_from_pollution_multiplier} * \text{birth_rate_from_material_multiplier}$ {people/year}

OUTFLOWS:

$\text{death_rate} =$

$\text{Population} * \text{normal_death_rate} * \text{death_rate_from_crowding_multiplier} * \text{death_rate_from_food_multiplier} * \text{death_rate_from_material_multiplier} * \text{death_rate_from_pollution_multiplier}$ {people/year}

$\text{birth_rate_from_crowding_multiplier} = \text{GRAPH}(\text{crowding_ratio})$

(0.000, 1.050), (1.000, 1.000), (2.000, 0.900), (3.000, 0.700), (4.000, 0.600), (5.000, 0.550)

$\text{birth_rate_from_food_multiplier} = \text{GRAPH}(\text{food_ratio})$

(0.000, 0.000), (1.000, 1.000), (2.000, 1.600), (3.000, 1.900), (4.000, 2.000)

$\text{birth_rate_from_material_multiplier} = \text{GRAPH}(\text{material_standard_of_living})$

(0.000, 1.200), (1.000, 1.000), (2.000, 0.850), (3.000, 0.750), (4.000, 0.700), (5.000, 0.700)

$\text{birth_rate_from_pollution_multiplier} = \text{GRAPH}(\text{pollution_ratio})$

(0.00, 1.020), (10.00, 0.900), (20.00, 0.700), (30.00, 0.400), (40.00, 0.250), (50.00, 0.150), (60.00, 0.100)

$\text{capital_fraction_indicated_by_food_ratio} = \text{GRAPH}(\text{food_ratio})$

(0.000, 1.000), (0.500, 0.600), (1.000, 0.300), (1.500, 0.150), (2.000, 0.100)

$\text{capital_investment_from_quality_ratio} =$

$\text{GRAPH}(\text{quality_of_life_from_material} / \text{quality_of_life_from_food})$

(0.000, 0.700), (0.500, 0.800), (1.000, 1.000), (1.500, 1.500), (2.000, 2.000)

$\text{capital_investment_generation_normal} = 0.05$ {capital units/person/year}

$\text{capital_investment_multiplier} = \text{GRAPH}(\text{material_standard_of_living})$

(0.000, 0.100), (1.000, 1.000), (2.000, 1.800), (3.000, 2.400), (4.000, 2.800), (5.000, 3.000)

$\text{capital_investment_ratio} = \text{Capital_Investment} / \text{Population}$ {capital units/person}

$\text{capital_investment_ratio_in_agriculture} =$

$(\text{capital_investment_ratio} * \text{Capital_investment_in_agriculture_fraction}) / \text{capital_investment_in_agriculture_fraction_normal}$ {capital units/person}

$\text{capital_investment_discard_normal} = 0.025$ {1/year}

$\text{capital_investment_in_agriculture_fraction_adjustment_time} = 15$ {years}

$\text{capital_investment_in_agriculture_fraction_normal} = .3$ {no units}

$\text{crowding_ratio} = \text{Population} / (\text{land_area} * \text{population_density_normal})$ {no units}

$\text{death_rate_from_crowding_multiplier} = \text{GRAPH}(\text{crowding_ratio})$

(0.000, 0.900), (1.000, 1.000), (2.000, 1.200), (3.000, 1.500), (4.000, 1.900), (5.000, 3.000)

$\text{death_rate_from_food_multiplier} = \text{GRAPH}(\text{food_ratio})$

(0.000, 30.000), (0.250, 3.000), (0.500, 2.000), (0.750, 1.400), (1.000, 1.000), (1.250, 0.700), (1.500, 0.600), (1.750, 0.500), (2.000, 0.500)

$\text{death_rate_from_material_multiplier} = \text{GRAPH}(\text{material_standard_of_living})$

(0.000, 3.000), (0.500, 1.800), (1.000, 1.000), (1.500, 0.800), (2.000, 0.700), (2.500, 0.600), (3.000, 0.530), (3.500, 0.500), (4.000, 0.500), (4.500, 0.500), (5.000, 0.500)

$\text{death_rate_from_pollution_multiplier} = \text{GRAPH}(\text{pollution_ratio})$

(0.00, 0.92), (10.00, 1.30), (20.00, 2.00), (30.00, 3.20), (40.00, 4.80), (50.00, 6.80), (60.00, 9.20)

$\text{effective_capital_investment_ratio} = (\text{capital_investment_ratio} * (1 - \text{Capital_investment_in_agriculture_fraction}) * \text{natural_resource_extration_multiplier}) / (1 - \text{captial_investment_in_agriculture_fraction_normal})$ {capital units/person}
 $\text{effective_captial_investment_ratio_normal} = 1$ {capital units/person}
 $\text{food_coefficient} = 1$ {no units}
 $\text{food_from_crowding_multiplier} = \text{GRAPH}(\text{crowding_ratio})$
(0.000, 2.400), (1.000, 1.000), (2.000, 0.600), (3.000, 0.400), (4.000, 0.300), (5.000, 0.200)
 $\text{food_from_pollution_multiplier} = \text{GRAPH}(\text{pollution_ratio})$
(0.00, 1.020), (10.00, 0.900), (20.00, 0.650), (30.00, 0.350), (40.00, 0.200), (50.00, 0.100), (60.00, 0.050)
 $\text{food_normal} = 1$ {food units/person/year}
 $\text{food_potential_from_capital_investment} = \text{GRAPH}(\text{capital_investment_ratio_in_agriculture})$
(0.000, 0.500), (1.000, 1.000), (2.000, 1.400), (3.000, 1.700), (4.000, 1.900), (5.000, 2.050), (6.000, 2.200)
 $\text{food_ratio} =$
 $(\text{food_coefficient} * \text{food_from_crowding_multiplier} * \text{food_from_pollution_multiplier} * \text{food_potential_from_capital_investment}) / \text{food_normal}$ {no units}
 $\text{land_area} = 135\text{e}6$ {square kilometers}
 $\text{material_standard_of_living} =$
 $\text{effective_capital_investment_ratio} / \text{effective_captial_investment_ratio_normal}$ {no units}
 $\text{natural_resource_extration_multiplier} = \text{GRAPH}(\text{natural_resource_fraction_remaining})$
(0.000, 0.000), (0.250, 0.150), (0.500, 0.500), (0.750, 0.850), (1.000, 1.000)
 $\text{natural_resource_fraction_remaining} = \text{Natural_Resources} / \text{natural_resources_initial}$ {no units}
 $\text{natural_resource_from_material_multiplier} = \text{GRAPH}(\text{material_standard_of_living})$
(0.00, 0.000), (1.00, 1.000), (2.00, 1.800), (3.00, 2.400), (4.00, 2.900), (5.00, 3.300), (6.00, 3.600), (7.00, 3.800), (8.00, 3.900), (9.00, 3.950), (10.00, 4.000)
 $\text{natural_resource_usage_normal} = 1$ {natural resource units/person/year}
 $\text{natural_resources_initial} = 900\text{e}9$ {natural resources units}
 $\text{normal_birth_rate} = .04$ {1/year}
 $\text{normal_death_rate} = 0.028$ {1/year}
 $\text{pollution_absorption_time} = \text{GRAPH}(\text{pollution_ratio})$
(0.00, 0.60), (10.00, 2.50), (20.00, 5.00), (30.00, 8.00), (40.00, 11.50), (50.00, 15.50), (60.00, 20.00)
 $\text{pollution_from_capital_multiplier} = \text{GRAPH}(\text{capital_investment_ratio})$
(0.000, 0.050), (1.000, 1.000), (2.000, 3.000), (3.000, 5.400), (4.000, 7.400), (5.000, 8.000)
 $\text{pollution_normal} = 1$ {pollution/person/year}
 $\text{pollution_ratio} = \text{Pollution} / \text{pollution_standard}$ {no units}
 $\text{pollution_standard} = 3.6\text{e}9$ {pollution units}
 $\text{population_density_normal} = 26.5$ {people/square kilometer}
 $\text{quality_of_life} =$
 $\text{quality_of_life_standard} * \text{quality_of_life_from_material} * \text{quality_of_life_from_crowding} * \text{quality_of_life_from_food} * \text{quality_of_life_from_pollution}$ {satisfaction units}
 $\text{quality_of_life_from_crowding} = \text{GRAPH}(\text{crowding_ratio})$
(0.000, 2.000), (0.500, 1.300), (1.000, 1.000), (1.500, 0.750), (2.000, 0.550), (2.500, 0.450), (3.000, 0.380), (3.500, 0.300), (4.000, 0.250), (4.500, 0.220), (5.000, 0.200)
 $\text{quality_of_life_from_food} = \text{GRAPH}(\text{food_ratio})$
(0.000, 0.000), (1.000, 1.000), (2.000, 1.800), (3.000, 2.400), (4.000, 2.700)
 $\text{quality_of_life_from_material} = \text{GRAPH}(\text{material_standard_of_living})$
(0.000, 0.200), (1.000, 1.000), (2.000, 1.700), (3.000, 2.300), (4.000, 2.700), (5.000, 2.900)
 $\text{quality_of_life_from_pollution} = \text{GRAPH}(\text{pollution_ratio})$
(0.00, 1.040), (10.00, 0.850), (20.00, 0.600), (30.00, 0.300), (40.00, 0.150), (50.00, 0.050), (60.00, 0.020)

```
quality_of_life_standard = 1 {satisfaction units}
{ The model has 60 (60) variables (array expansion in parens).
  In root model and 0 additional modules with 7 sectors.
  Stocks: 5 (5) Flows: 8 (8) Converters: 47 (47)
  Constants: 16 (16) Equations: 39 (39) Graphicals: 22 (22)
}
```