# Model 7: Exogenous Economic Growth

**Narrative**

Humanity did not heed Malthus' warning and curb its population growth, but, to the surprise of those demographers who followed Malthus, not only did human population keep on expanding without some sort of crash, but the material life of the producers (many of them now laborers in the factories power by coal-guzzling steam engines) got better, not worse.

Contrary to Malthus' prediction, during the 19th century the growth of technology in the industrializing part of the world led to economies that grew even faster that the human population. Instead of crashing against subsistence limits as had been the lot of humans throughout the Holocene, life got better and better—even for the farmers and the new class of industrial workers. Economic historians celebrate this event as the first time in human history when the lot of almost everyone increased for generation after generation, riding on the back of a never-ending technological and social revolution.

In the minds of many social scientists, especially economists, the industrial "revolution" is one of the most important events in human history. In one sense this is certainly true. Malthus was a good empirical scientist and knew that (at least up until the end of his life in 1834) the rate of technological progress had been slower than the potential rate of population growth. Even if the carrying capacity of a country was rising, population bumped along close behind, rapidly taking up all the slack between population size and carrying capacity. Hunger, disease, sexual abstinence, and other unfortunately painful forces kept the population in check. Thus, misery was conserved. Even after the waves of Plague that cut the Old-World population by a third to a half beginning in the 14th century, populations closed back in on the slowly rising carrying capacity in about a century.

The data available to Malthus were limited. He, of course, had no data on the last 2/3rds of the 19th century and the data of the time extended back only a few centuries. Thus, Malthus and his contemporaries had no way of understanding what we see as the agricultural conflagration. Over short periods of time, even the few centuries that Malthus knew, the growth of a building conflagration looks quite linear. But the rate of technological innovation had been very slowly growing if our conflagration model is correct. At some point the rate of technological progress was going to exceed the capacity of the population to grow and it was happening right under Malthus' nose! He was 68 when he died. If Malthus had lived two or three more decades, he would have seen the pattern. The quickening pace of the technological progress was first noticeable in Britain but rapidly spread to the rest of Northwestern Europe and with a lag of decades (Japan) or a century and a bit (China, India) to the advanced Far Eastern societies that had previously had turns being leaders in technological evolution. By now the rate of technological progress is high in most countries. The fire of the agricultural conflagration is now burning very hot causing problems that we'll analyze in our final model.

The literature on the European industrial revolution is vast. Much of it has the celebratory Rise-of-the West character that doesn't take account of the fact that the Western Industrial Revolution rested on innovations from other parts of the world, some ancient like the wheel, and some, like the printing press and the canal lock, relatively recent Chinese innovations. What does seem to be clear is that innovation is highly localized in time and space. The various regions of the Old World took turns at leading innovation. Africa was the locus of the cultural and genetic evolution of humans that left Africa about 50,000 years ago. Eventually our species became the only living species of our genus, pioneering previously uninhabited regions like Australia and the New World. The Near East was the most productive region for plant and animal domesticates in the Early Holocene and the region where states first evolved in the Middle Holocene. In the early Islamic period, the region was, intellectually, the most advanced society, preserving much of the Classical knowledge developed by the ancient Mediterranean societies like Greece and Rome while making advances of their own in many fields. During the European Dark and Middle Ages, India and China were the most innovative societies. The innovations from one center diffused to the others, forming the launching pad for the next center of innovation to take off from.

Within the generally innovative regions, innovation could be highly localized. The small early industrial district of Ironbridge in the West Midlands of England took advantage of a deep, narrow, gorge of the Severn River that exposed deposits of iron ore, pottery clay, limestone, and coal, to establish a suite of pioneering and innovative industrial firms. The electronics and computer firms in California's Silicon Valley were the source of a disproportionate number of innovations in these fields. The great research universities, Berkeley and Stanford, contributed renowned physicists and engineers and well-trained industrial R&D professionals.

A legendary firm, Fairchild Semiconductor was formed in 1957 by former associates of transistor pioneer and Stanford professor William Shockley. The firm revolutionized the production of conventional transistors and went on to develop the integrated circuit that put many transistors on a single silicon chip, perhaps as important as the transistor itself to the electronics revolution. The company was a great financial success at first, but its most important product in the end was the talent it nurtured. The Fairchild talent dispersed to other ambitious firms and innovative startups such as the massively successful chipmaker Intel for example. In the meantime, a powerful venture capital sector formed in Silicon Valley that was willing to fund promising but risky startup companies, turbocharging innovation. The Economist Brian Arthur has advanced accident-plus-positive feedback models for why that innovation process is unpredictable and localized.

We have no idea if ancient and medieval centers of innovation resembled those of the Industrial Revolution or not. In keeping with the conflagration model, the more recent bursts of innovation are likely to have been faster, as the electronics revolution was faster than the coal-iron-steel-textiles revolution. Certainly the diffusion of innovations has accelerated with navigational improvements beginning in the 15$^{th}$ century and telecommunications innovations beginning in the 19$^{th}$ century. The comparative study of innovative revolutions should yield interesting dividends!

Here we reprise the MIT economist Robert Solow's 1956 model of the growth of economic output as a function of labor, capital, and, critically, technology. It was a pioneering use of dynamic analysis in economics. Interestingly, he was a PhD student of Wassily Leontief, who was responsible for another pioneering application of dynamic models, input-output analysis. Solow assumed that labor and technology increased exponentially at exogenously given rates (assumed rather than generated internally by the model), and that some portion of output was saved to create new capital. In his famous model, presented in this unit, model growth per capita converges on a steady growth path determined by the rate of technological progress.

In 1957, Solow published a paper applying his model to the analysis of data. Economists had time series for the accumulation of capital, the growth in the labor supply and aggregate economic output. They did not have a measure of technological progress. Solow asked if you take labor and capital growth into account, what is left over to be explained by technological progress? The answer turned out to be most of it! About 80% of the per capita economic growth in the US was attributable to technological progress and only about 20% to the growth of capital per worker. This finding led policy makers to pay attention to supporting research and development rather than just capital investment. Solow's finding was also important in environmental economics because it implied that technological improvement could reduce the impact of economic activity on the environment by making capital work more efficiently, for example by reducing the amount of $CO_2$ produced per unit of economic output.

Solow's model treated the growth of technology as being exogenous, i.e., as resulting from external factors. Solow won the 1987 Nobel Prize in economics for this model. Many an ambitious economist must have thought to themselves "how stupid of me not to have thought of that" (Thomas Henry Huxley, Darwin's bulldog, said this about Darwin's theory of natural selection). Many of the greatest scientific discoveries turn out to be surprisingly simple, pointing clearly to the heart of a major problem, a corollary of the KISS principle.

## Further Reading


Griliches, Z. (1957). "Hybrid corn: An exploration of the economics of technological change." *Econometrica* 48: 501-522.

Hobson, J. M. (2004). *The Eastern Origins of Western Civilization.* Cambridge University Press.

Lindert, P. H. (1985). "English population, wages, and prices:1541-1913." *Journal of Interdisciplinary History* 15: 609-634.

Solow, R. M. (1956). "A Contribution to the Theory of Economic Growth." *The Quarterly Journal of Economics* 70(1): 65-94.

Solow, R. M. (1957). "Technical change and the aggregate production function." *The Review of Economics and Statistics* 39(3): 312-320.

Stearns, P. N. (2020). *The Industrial Revolution in World History.* Routledge.

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https://en.wikipedia.org/wiki/Great_Divergence
https://en.wikipedia.org/wiki/The_Eastern_Origins_of_Western_Civilisation
https://en.wikipedia.org/wiki/Economic_growth
https://en.wikipedia.org/wiki/Robert_Solow
https://en.wikipedia.org/wiki/W._Brian_Arthur


## White Box Graphical Model

The under-the-hood model description sections can be skipped, and you can proceed directly to the Black Box Simulations below if you just want to operate the simulator and skip the model diagram and equations.

The graphic Stella model shown in Figure 7-1 (below) is broken into two four sections: Capital, Economics, Technology, and Labor.

Model 7: Exogenous Economic Growth

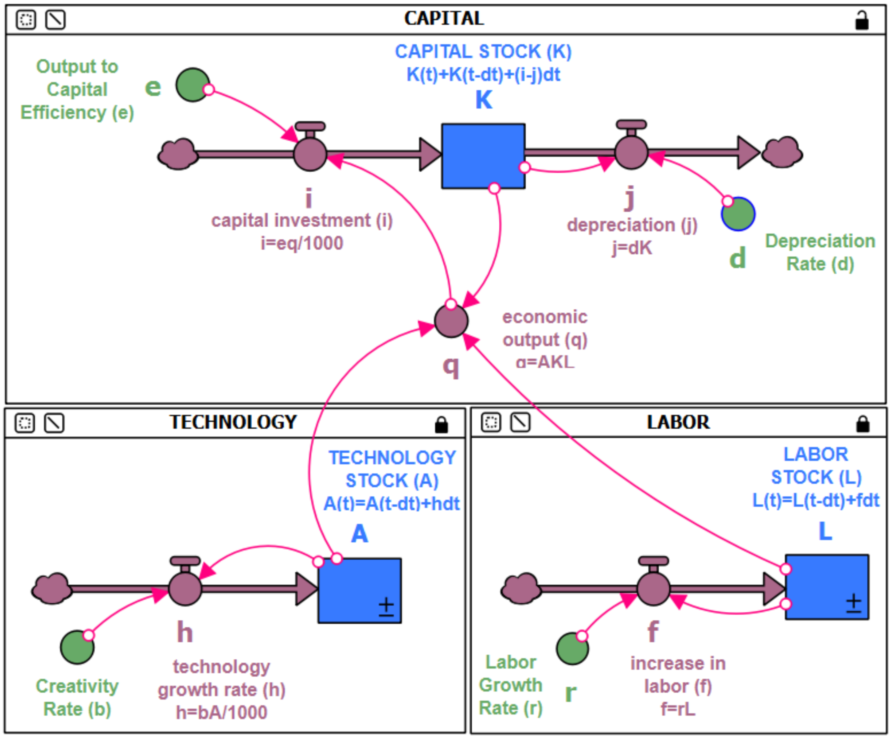


*Figure 7-1: Exogenous Economic Growth Model visual flow diagram*

*Model Variables and Equations*

**Key:** STOCKS, Parameters, and intermediate variables

| CAPITAL | Units | Stella Equations |
|---|---|---|
| CAPITAL STOCK (K) | | K(t) = K(t-dt) + (i-j)dt |
| Output to Capital Efficiency (e) | | |
| Depreciation Rate(d) | | |
| capital investment (i) | | i=eq/1000 |
| depreciation (j) | | j=dK |
| economic output (q) | | q=AKL |
| | | |
| **TECHNOLOGY** | | |
| TECHNOLOGY STOCKS (A) | | A(t) = A(t-dt) + hdt |
| Creativity Rate (b) | | |
| technology growth rate (h) | | h=bA/1000 |
| | | |
| **LABOR** | | |
| LABOR STOCK (L) | | L(t) = L(t-dt) + fdt |
| Labor Growth Rate (r) | | |
| Increase in labor (f) | | f=rL |

*Table 7-1: Model variables and equations.*

The Exogenous Economic Growth Model visual flow diagram “white box” model can be reduced to a set of initial conditions and independent (and intermediate) variables which, through mathematical relationships (equations) provide the results (the independent variables).

*Equations without Intermediate Variables*

K' = eAKL-dK

A' = bA

L' = rL

**Black Box Simulations**

When using a black box model, one is just concerned with the model’s inputs, not its internal workings which can be complex. To run this model from this black box perspective, bring it up at

https://exchange.iseesystems.com/public/cherylgenet/exogenous-economic-model/index.html#page1

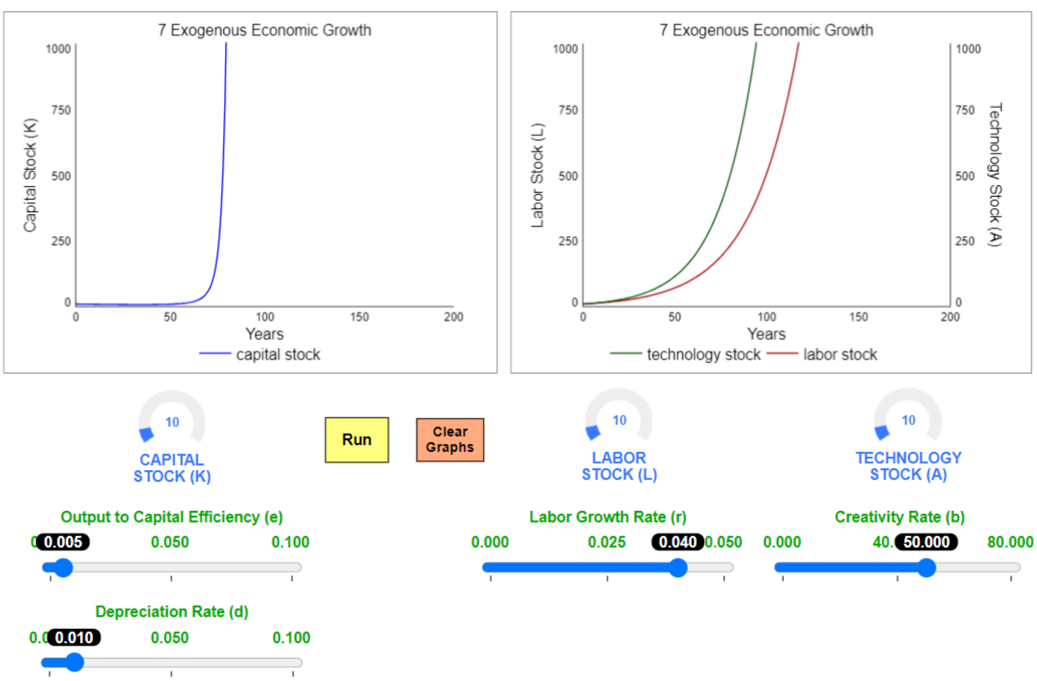


*Figure 7-2: Base Exogenous Economic Growth model.*

Your model has seven controls, three initial condition knobs and four independent variable sliders.

Initial condition knobs:

- Capital Stock (K)
- Labor Stock (L)
- Technology Stock (A)

Independent variable sliders:

- Output to Capital Efficiency (e)
- Depreciation Rate (d)
- Labor Growth Rate (r)
- Creativity Rate (b)

The initial condition knobs and independent parameter sliders require minimum, maximum, increment (resolution), and reset values. These are provided in the table below.

**Key:** STOCKS, Parameters

| | Min | Max | Increment | Reset |
|---|---|---|---|---|
| **CAPITAL** | | | | |
| CAPITAL STOCK (K) | 0.0 | 100.0 | 1.0 | (10) |
| Output to Capital Efficiency (e) | 0.0 | 0.1 | 0.01 | 0.005 |
| Depreciation Rate(d) | 0.0 | 0.1 | 0.01 | 0.01 |
| | | | | |
| **TECHNOLOGY** | | | | |
| TECHNOLOGY STOCKS (A) | 0.0 | 100.0 | 1.0 | (10) |
| Creativity Rate (b) | 0.0 | 80 | 0.001 | 50 |
| | | | | |
| **LABOR** | | | | |
| LABOR STOCK (L) | 0.0 | 100.0 | 1.0 | (10) |
| Labor Growth Rate (r) | 0.0 | 0.05 | 0.01 | 0.04 |
| | | | | |
| **OUTPUT GRAPHS 1** | | | | |
| capital stock | 0.0 | 1000.0 | 1.0 | -- |
| | | | | |
| **OUTPUT GRAPHS 2** | | | | |
| technology stocks | 0.0 | 1000.0 | 1.0 | -- |
| labor stocks | 0.0 | 1000.0 | 1.0 | -- |
| | | | | |
| years (t) | 0.0 | 200.0 | 1.0 | -- |

*Table 7-2: The simulator settings for Model X: Model Name*

*Basic Economic Growth Scenario*

The growth of technology in this exogenous model is set by the Creativity Rate (b), while the growth of the CAPITAL SOCK (K) is sensitively affected by the Output to Capital Efficiency (e). With these parameters set relatively low, the certain, eventual rapid growth toward infinity of this model is delayed. In this first scenario, the delay matches, roughly, what has happened in the West since the beginning of the industrial revolution. This suggests that, for much of the industrial revolution the creativity rate was low, as was the investment in capital efficiency (such as research and development). This first scenario is shown above in Figure 7-2 so will not be repeated here. It is the default settings for this model.

*Rapid Economic Growth Scenario*

On the other hand, if the Creativity Rate (b), and the Output to Capital Efficiency (e) are set relatively high, the rapid growth toward infinity of this model is occurs much sooner. Now that our creativity rate is high, and we are purposely investing in research and development, this rapid growth might be suggestive of our current situation with respect to the future.

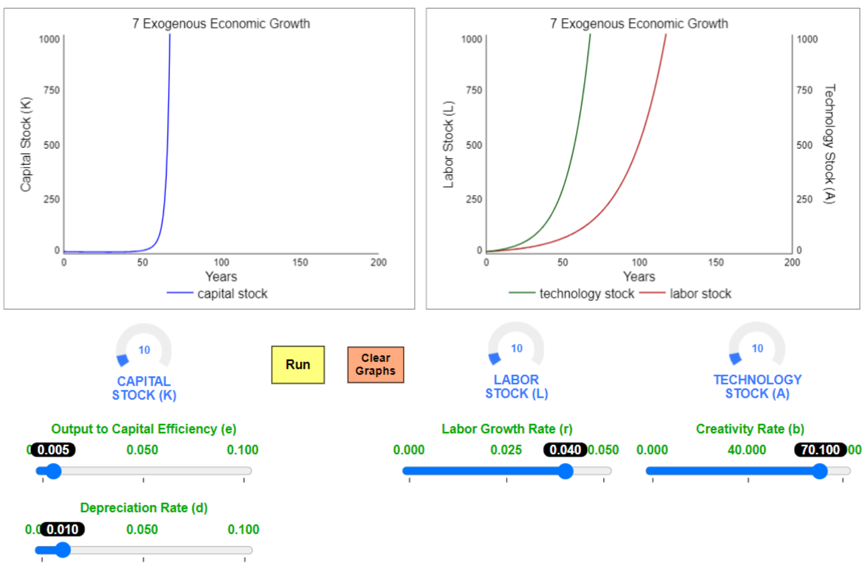


*Figure 7-2: Rapid economic growth scenario.*

**Conclusions**

Solow's exogenous model—and the data that backed it up—demonstrates that the spectacular growth of industrial economies that outpaced population growth (thus escaping a Malthusian treadmill) was due in a large part to technological advances and production efficiencies, not just to increasing capital and labor stock as had been previously assumed.

**Appendix / Stella Top Level Model Code**

A Stella model is created by connecting the graphical elements and entering information in the Stella GUI interface. Once everything is connected and entered, Stella automatically creates the "top level code." This code provides a good check on whether or not the Stella model is what you really intended and can be very useful in trouble shooting models that are not providing reasonable results or don't seem to be working at all.

```
Top-Level Model:
A(t) = A(t - dt) + (h) * dt
   INIT A = 10
   INFLOWS:
      h = A*b/1000 {UNIFLOW}
K(t) = K(t - dt) + (i - j) * dt {NON-NEGATIVE}
   INIT K = 10
   UNITS: biomass unit
   INFLOWS:
      i = e*q/1000 {UNIFLOW}
         UNITS: biomass unit/years
   OUTFLOWS:
      j = K*d {UNIFLOW}
         UNITS: biomass unit/years
L(t) = L(t - dt) + (f) * dt
   INIT L = 10
   INFLOWS:
      f = L*r {UNIFLOW}
b = 50
d = .01
e = 0.005
q = A*K*L
r = 0.04
{ The model has 12 (12) variables (array expansion in parens).
  In root model and 0 additional modules with 3 sectors.
  Stocks: 3 (3) Flows: 4 (4) Converters: 5 (5)
  Constants: 4 (4) Equations: 5 (5) Graphicals: 0 (0)
  }
```