

Model 1: Population Growth

Narrative

The first model, Population Growth, is a purposely simple model that introduces the Stella graphic model diagram (flow model) symbology and terminology, and also introduces the first online model interface. For the mathematically inclined, the model's single mathematical equation is provided, although mathematical knowledge is not required to understand and run the model.

However, in spite of its simplicity, the insight from this model is not trivial. Populations have an inherent tendency to rapidly expand even if this tendency cannot go on unmodified for long. Hence, however much complexity is added to population models, an exponential growth term is usually part of the equations—often the heart of them.

Imagine a scenario in which a few human hunter-gatherer families stepping off a raft on a deserted but lush tropical island with unlimited food. This simple model examines how their population would quickly soar if there were no restraints. In reality, as Thomas Malthus suggested, an exponential growth would lead to humanity outstripping any food supply.

Thomas Malthus (1766-1834) is considered by many to be the father of demography, the study of human populations—their size, composition, and distribution. Births, deaths, and migration are the primary drivers of population changes. He also much influenced ecologists and evolutionary biologists interested in population behavior more generally. For example, Charles Darwin's idea of natural selection came to him while contemplating Malthus' argument about the inevitability of environmental limits to population growth.

Malthus gathered data on population growth from many countries and concluded that most populations were growing *exponentially*. Some populations were doubling almost every generation, so from one generation to the next it was growing at the rate of 1, 2, 4, 8, 16, ..., a growth rate that is exponential. It is easy to mathematically model exponential growth.

The basic biology of reproduction leads to exponential growth. Each pair of parents has children who in turn become parents. So long as each pair of parents on average has more than two offspring who survive to become parents, the population will grow exponentially, if less than two, it will decline exponentially. In recent popular discourse, "exponentially" has come to be a synonym for "fast," but that does not do justice to the mathematical insight that exponential processes *accelerate* or *decelerate* with time. If the number of offspring is only slightly more than two, exponential growth will be very slow for a long time, but eventually take off toward infinity like a rocket.

On the other hand, Malthus suggested that consumable resources (food) was only growing at a rate of 1, 2, 3, 4, 5, ..., a growth rate that is linear (constant) rather than increasing with time. His point was that even if a linear growth rate is at first much faster than an exponential one, eventually an accelerating exponential rate will catch up and then exceed the linear one.

In 1798 Malthus published his population growth data on a number of countries along with his theory as an anonymous book, *An Essay on the Principle of Population*, with a subtitle *As it Affects the Future Improvement of Society*. The inspiration for his book came from the clash between his data (rapidly

rising populations) and the optimistic views about the future “perfection” of humanity held by Rousseau and others. Malthus was soon identified as the author, and the book quickly sold out amid great controversy. A second edition of Malthus’ book came out in 1803, and a final, sixth edition in 1826.

Malthus maintained that an exponentially growing population would, inevitably, lead to a rising supply of labor which, in turn, would result in lower wages. He knew that this had, in fact, happened in the past. In the aftermath of the Black Death in the 14th century, the labor supply contracted sharply in Britain. Wages rose and farm tenancies went begging, depressing rents. But rapid population increase in only a few generations quickly made up the deficit. Wages fell, rents rose, and the working classes’ standard of living went down.

Malthus proposed two solutions to this problem. One was to decrease the birth rate through late marriage and abstinence. He knew that there were practices besides abstinence to avoid pregnancy, but he considered these sinful. If the birth rate did not reduce sufficiently, Malthus suggested rather graphically, that the second solution—an increased death rate—would be inevitable.

The power of population is so superior to the power of the earth to produce subsistence for man, that premature death must in some shape or other visit the human race. The vices of mankind are active and able ministers of depopulation. They are the precursors in the great army of destruction, and often finish the dreadful work themselves. But should they fail in this war of extermination, sickly seasons, epidemics, pestilence, and plague advance in terrific array, and sweep off their thousands and tens of thousands. Should success be still incomplete, gigantic inevitable famine stalks in the rear, and with one mighty blow levels the population with the food of the world.

Malthus suggested that providing welfare to the poor (via the somewhat humane Poor Laws then in effect in England) would, in the long run, only make matters worse for the lower class. The business oriented Whig party liked Malthus’s suggestion and eventually passed a law that stipulated that no poor person was to receive any money or help unless they labored in a workhouse. By purposely making the food and living conditions in the workhouses quite terrible and the pay lower than the pay offered anywhere else, the poor would try hard to obtain gainful employment elsewhere. In practice, many people starved rather than go to a workhouse, so the workhouses were, in a grotesque way, effective. They also provided a very low-cost source of near slave labor and created the conditions reported on so effectively by Charles Dickens. In *A Christmas Carol*, Dickens has Scrooge say:

I don't make merry myself at Christmas and I can't afford to make idle people merry. I help to support the establishments [the workhouses] I have mentioned: they cost enough: and those who are badly off must go there. Many can't go there; and many would rather die. If they would rather die," said Scrooge, "they had better do it, and decrease the surplus population.

The idea that the poor need to be driven to work by the threat of hard want has not entirely disappeared since Dickens’ day!

Malthus was an important pioneer. His model is extremely simple, but it captures a basic bit of biological reality. In the two centuries since Malthus, ecologists and demographers have added new elements to his model of exponential growth, but, as you will see in the following models, it still lives inside successor models in one form or another.

As mentioned in the Introduction, one can learn a lot about how models behave just by running them. If you would like to drive a model and skip looking under the hood first, go directly to Black Box Model below. You can always come back to this point and visit the sausage factory.

Further Reading

Lindert, P. H. (1985). "English population, wages, and prices:1541-1913." *Journal of Interdisciplinary History* 15: 609-634.

https://en.wikipedia.org/wiki/Thomas_Robert_Malthus

https://en.wikipedia.org/wiki/Exponential_growth

https://en.wikipedia.org/wiki/System_dynamics

White Box Graphical Model

While you can learn how simulation models behave just by running them as black box, just as you can learn how cars behave by just driving them, additional insights about models are gained when you first look at a model's visual diagrams, i.e., when you look inside the white box or under the hood. Also, if you plan on eventually building your own models, exploring the details of existing models will provide you with a helpful background.

This section introduces you to the conventions of the Stella simulation engine. Once you master these conventions you will find that you can understand the structure of quite complex models from either the Stella graphical representation or the mathematical equations. Some people find the visual representation easier to follow than the abstract equations. Equations were invented to make algebra easier, but since we are not going to require you to do any algebra, they are not essential for this module. If you are interested in modifying the models we present here, you can use values listed in the Stella graphical interface table to assist you, or you can use the equations in conjunction with other simulation engines you may already know how to use.

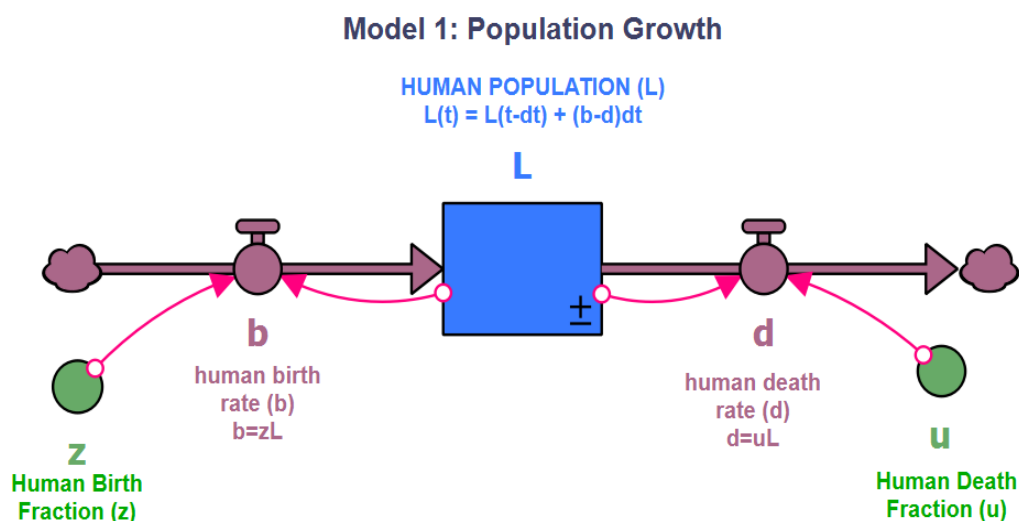
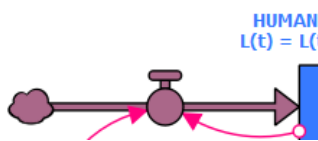


Figure 1: Population Growth model diagram.

The components in the population model shown above are as follows:



The rectangular box is the *stock*, which is some quantity that is accumulated or lost over time. In this case it is the number of people, i.e., the population. You can think of it as a tank or bathtub that can have different levels of water over time depending on how much water is flowing in and flowing out. The number of people (or water level) depends on the past history of both the inflow and outflow. Not shown in the model, but assumed, is that there is some *initial value* assigned to the number of people (or gallons of water) in the stock box (the water tank).



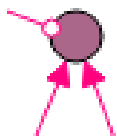
The “cloud,” the “faucet,” and double line represent the “flow” into the stock (tank). The cloud is the *source* of the potential people, assumed of infinite size (the model will never run out of a source of new people). The faucet (or valve) controls the *flow* of the people from the source to the stock. The wide arrow at the end of the flow indicates the allowed direction of the flow (just into the stock in this case).



A *connector*, which has a small circle on one end and an arrow on the other end, shows a flow of information which denotes action, with the arrow providing a cause and effect direction. The small arrows pointing at the valve (above), for instance, are information flows that control the setting of the valve. Not only does the valve regulate (meter) the flow, but it does so based on one or more information inputs. In the case of multiple inputs, the valve will always include some simple arithmetic function such as multiplication that is performed on the inputs to control the flow.



Green circles, which only have an output and no inputs, are simply the input setting of some parameter value to the model, i.e., some independent variable. These values, which the model user is able to change, are entered via the black box interface sliders.



Finally, mauve circles, which have both inputs and outputs, are *converters* where some simple arithmetic function such as multiplication is performed on the inputs to produce the output. The simple

population model does not have a converter. However, the valves, described above, also function as converters. The output of a converter (or valve) is sometimes called an “intermediate variable” because it is between the independent variable model inputs and the final model outputs, the dependent variables (the results, i.e., the plots of the dependent variables versus time).

In some ways, intermediate variables break up the model into more easily understood chunks, but in the process, they also complicate the model. While intermediate variables are almost always included in Stella models, they are rarely used in pure mathematical representations—considered an unnecessary complication. If one starts with a set of equations that include intermediate variables, one can eliminate them via successive substitution, ending up with equations that only include independent and dependent variables.

Model Variables and Equations

The Population Growth visual flow diagram “white box” model can be reduced to a set of initial conditions and independent (and intermediate) variables which, through mathematical relationships (equations) provide the results (the dependent variables).

Variable	Units	Model Equation
HUMAN POPULATION (L)	People	$L(t) = L(t-dt) + (b-d)dt$
Human Birth Fraction (z)	1/year	
Human Death Fraction (u)	1/year	
human birth rate (b)	People/year	$b = zL$
human death rate (d)	People/year	$d = uL$

Table 1-2: Model equations.

The table provides, for the Population Growth model, the units and equation (where appropriate) of the **DEPENDENT VARIABLE** (result), the **Independent Variables** that can be set by the model user, and the **intermediate variables** that establish relationships.

The basic model equation is: $L(t) = L(t-dt) + (b-d) dt$

Where:

$L(t)$ is the human population (units of people) at some time t

$L(t-dt)$ is the human population at some slightly earlier time $(t-dt)$, where dt is a small increment of time

b and d are the birth and death rates in units of people/year.

When $(b-d)$ in people/year is multiplied by time dt in years, the result (units) is people, the number of people to be added to or subtracted from the total **HUMAN POPULATION (L)** for the increment of time dt .

However, b and d are intermediate variables which are calculated from independent variables, so we need two additional equations to make these calculations: $b = zL$ and $d = uL$, where z and u are the human birth and death fractions of all the people that are born or die in a population in one year. The units for this are people (who are born or die)/people (total population)/year, or simply 1/year.

We can now replace the intermediate variables, b and d, with the independent variables, z and u, to come up with the equation we are really looking for, one that relates the dependent (output) variable to the independent (input) variables:

$$L(t) = L(t-dt) + (zL - uL) dt$$

or, collecting terms,

$$L(t) = L(t-dt) + (z-u)L dt$$

While this provides us with the value of the independent variable, L, at time t, given that we have the value of L at (t-dt), what we really want is the value of L at time t *without* having to know the value at an earlier time. To do this we need to run the simulation or perform the mathematical equivalent, i.e., make the integration:

$$L(t) = L_0 + \int (z-u)L dt$$

Where

L_0 is the initial population at the start, $t = 0$

$\int$ stands for the integral from $t = 0$ to the present (t)

For most dynamic system models, a numerical integration with either a computer or a room full of computer ladies with calculators is required to calculate $L(t)$. However, for very simple models like this one, an analytic solution exists. In this case the solution is:

$$dL/dt = (z-u)L \text{ and hence}$$

$$L(t) = L_0 e^{(z-u)t} \text{ where } e \text{ is the base of the natural logarithms}$$

Black Box Model

As suggested in the course Introduction, when using a black box model, one is just concerned with the model's inputs, not its internal workings which can be extraordinarily complex. To run the Population Growth model from this black box perspective, bring it up at

<https://exchange.iseesystems.com/public/cherylgenet/population-growth>

This is what you should get:

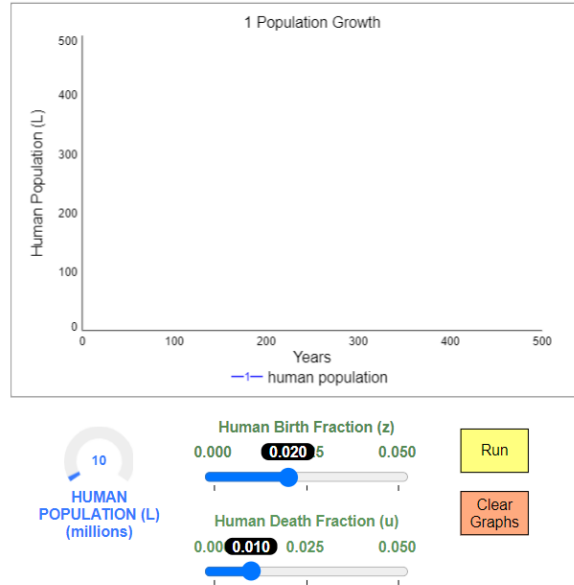


Figure 2: Initial model before running.

Your model has five controls. These are:

- Human Population (millions) knob (initial condition)
- Human Birth Fraction slider (independent variable)
- Human Death Fraction slider (Independent variable)
- Run button
- Clear Graphs button.

	Min	Max	Increment	Reset
HUMAN POPULATION (L)	0	300	1	10.0
Human Birth Fraction (z)	0	0.05	0.001	0.02
Human Death Fraction (u)	0	0.05	0.001	0.01
OUTPUT GRAPHS				
HUMAN POPULATION (L)	0	500		
years (t)	0	500		

Table 1-1: The simulator settings for Model 1: Population Growth

Each simulator control has a minimum and maximum value. These values cannot be changed by the model user and have been set by the model designers to allow the model to be exercised over a useful range of values while avoiding extreme values that would be unrealistic.

In considering how populations grow (or shrink), demographers employ *birth rates* and *death rates*. A birth rate is defined as the number of births per 1000 individuals per year in the population. The “individuals” include all women and men of all ages—everyone. The “per 1000” provides a number larger than 1 instead of a fraction. The *birth fraction* (used in this Stella model) is the number of births per individual per year and is thus 1000 times smaller than the birth rate. The *death fraction* is, similarly, the number of deaths per individual per year.

When the Population Growth Model comes up it will have these default settings (as shown in the table above):

HUMAN POPULATION (L) = 10

Human Birth Fraction (z) = 0.02

Human Death Fraction (u) = 0.01

Now click the *Run* button and watch the output screen as the computer calculates the increasing population over time and plots out **blue curve 1**.

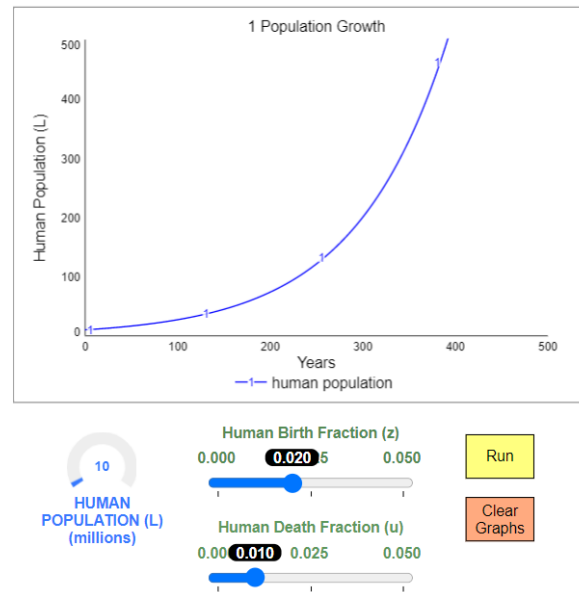


Figure 1-3: Simulation results.

Note that the population, besides increasing as time goes by, is increasing faster and faster, i.e. population growth is accelerating. The reason for this is that, in the model, the **human birth rate (b)** is a function of both the **Human Birth Fraction (z)** and the **HUMAN POPULATION (L)**. Mathematically, $b = zL$. Thus, as the population gets larger, the number of births keeps going up, the birth valve keeps getting opened wider and wider. Infinity here we come!

Questions:

What was the initial population? Easy, just look at the table above: 10

What was the population after 100 years? To find this, put your cursor on the graph and hold down the left button. Move the vertical line until Years = 100. Right below that is 27, the population in Year 100.

Was the birth fraction greater than the death fraction? Yes, just look at either the default table or the values on the two independent variable sliders, **Human Birth Fraction (z) = 0.020** and **Human Death Fraction (u) = 0.010**.

How long did it take the population to double? Hold down the left mouse button. At Year 0 it reads 10. Now move the mouse-controlled vertical line to the right until the population just clicks to 20 and read below Year for the answer: ~70 years (answer can vary by a year or two).

Leaving all other controls alone, decrease the **Human Death Fraction (u)** until it reads 0.00, and then click *Run*.

With a smaller death fraction, the population increased even faster over time.

Question: Did the population change the way you expected it to before you pushed the *Run* button?

Now create a different scenario. Start with a large number of natives arriving on the deserted island and a death fraction that is *larger* than the birth fraction. What do you expect the results curve will look like? Now push *Run* and see what you get.

The shape of the curve reflects that when the population is the largest, the most people will die, but as the population falls, less people will be dying. According to the model, the population will never quite get to zero until time approaches infinity, although we know that in reality, once the population drops below two people the game is over. Our model is not exactly true, but it is useful approximation for many purposes. This is a good example of the expression that “all models are wrong, but some are useful.”

Now decrease the **Human Birth Fraction (z)** until it exactly matches the **Human Death Fraction (u)** and click *Run*. *Questions:* What kind of output curve did you get? Was it what you expected? What do you think would happen, in the long run, if you made just a tiny change in either the birth rate or the death rate?

Exercises:

Given a **HUMAN POPULATION (L)** of 10 can you, by adjusting the sliders via trial and error, find a combination of **Human Birth Fraction (z)** and **Human Death Fraction (u)** that will increase the population by a factor of ~ 40 in 200 years? Answer: Yes you can, but note that your answer will not be unique.

Conclusions

In the simple Malthusian population model, if the birth rate exceeds the death rate then, sooner or later, the population will take off and head toward infinity, blowing up the model. On the other hand, if the death rate exceeds the birth rate, eventually everyone dies, and the population drops to zero and stays there. Finally, if the birth and death rates are exactly equal, the population will stay the same, but this is an *unstable* equilibrium because the slightest change in either the birth rate or death rate will eventually cause the population to head toward infinity or zero.

Appendix / Stella Top-Level Model Code

A Stella model is created by connecting the graphical elements and entering information in the Stella interface. Once everything is connected and entered, Stella automatically creates the “top level code” which is shown below for Model 1. This code provides a good check on whether or not the Stella model is what you really intended, and can be useful in trouble shooting models that are not providing reasonable results or do not seem to be working at all.

Top-Level Model:

$L(t) = L(t - dt) + (b - d) * dt$

INIT L = 10

UNITS: Population

INFLOWS:

```

    b = L*z {UNIFLOW}
      UNITS: Population/Years
OUTFLOWS:
    d = L*u {UNIFLOW}
      UNITS: Population/Years
u = 0.01
  UNITS: per year
z = 0.02
  UNITS: per year
{ The model has 5 (5) variables (array expansion in parens).
  In root model and 0 additional modules with 0 sectors.
  Stocks: 1 (1) Flows: 2 (2) Converters: 2 (2)
  Constants: 2 (2) Equations: 2 (2) Graphicals: 0 (0)
}

```